12/2/2015

Dynamics of a particle

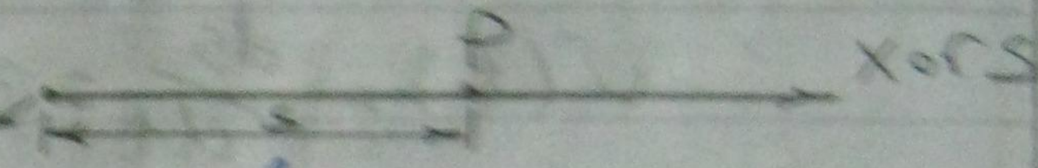
Kinematics — Kinetics

Kinematics of a particle:

- Properties of motion without taking reasons into consideration

* Rectilinear motion

- In figure:



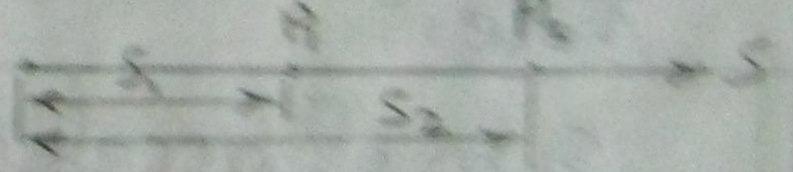
Particle move along an horizontal axis

s: distance from a fixed point (origin)

s: represents Position.

s is function of time as it varies with time $s(t)$

D: Displacement between time interval (t_1 to t_2)



Displacement: $D = \Delta S = S_2 - S_1$

D: represents difference in position.

Velocity: The rate of change of position with time.

$$v(t) = ds/dt$$

Acceleration: The rate of change of velocity with respect to time.

$$a(t) = dv/dt$$

S $\underset{\text{Integration}}{\overset{\text{Derivation}}{\rightleftarrows}}$ D $\underset{\text{Integration}}{\overset{\text{Derivation}}{\rightleftarrows}}$ a

* Jerk: change of acceleration with respect to time.

$$J(t) = \frac{da}{dt}$$

- It happens when acceleration changes Random not smooth.

Example $s(t) = t^3 - 9t^2 + 15t$ m

Find 1) Initial velocity and acceleration.
2) The Time at which the velocity vanishes

$v(t) = \frac{ds}{dt} = 3t^2 - 18t + 15$ (m/s) ($v=0$)

$a(t) = \frac{dv}{dt} = 6t - 18$ (m/s^2)

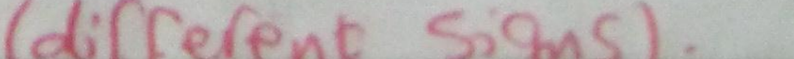
$J(t) = \frac{da}{dt} = 6.$ (m/s^3)

Answer 1) let $(T=0) \Rightarrow s_0 = 0$

$v_0 = 15\ m/s$, $a_0 = -18 m/s^2$

Note:

sign of acceleration: (-ve) → to left , (+ve) to right.



Type of acceleration ① → accelerating rate
② → decelerating rate.

→ accelerating is (+ve): when $\vec{v}$ and $\vec{a}$ in the same direction (The same sign).

→ decelerating is (-ve): when $\vec{v}$ and $\vec{a}$ in opposite directions (different signs).

→ momentrly stillness: The particle changes It's direction of motion at that moment.

Answer ② : let $v=0 \Rightarrow 3t^2 - 18t + 15 = 0 \ (\div 3)$

$$t^2 - 6t + 5 = 0$$

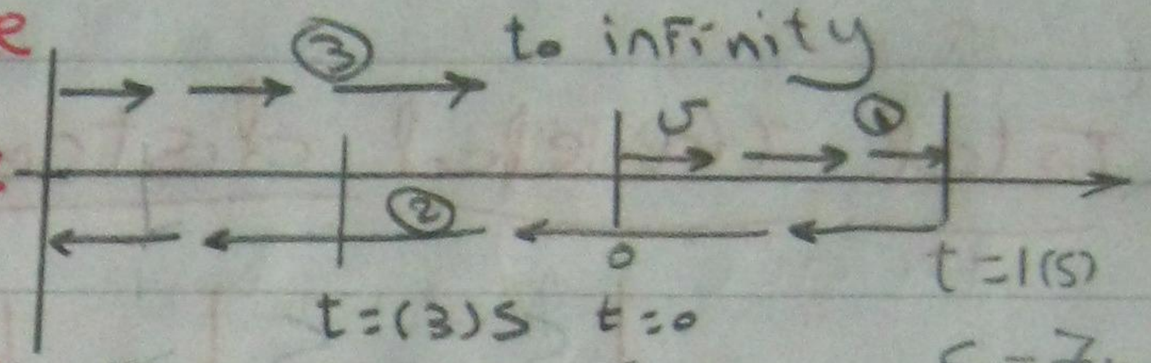
$$(t-5)(t-1) = 0$$

Hence, $t = 1$ (s), $t = 5$ (s)

At $\underline{T=1\,(s)}$ $S_1 = 7\,m$, $a_1 = -12\,m/s^2$.

At $\underline{T=5\,(s)}$ $S_2 = -25\,m$, $a_2 = 12\,m/s^2$.

① The particle moves along the (+ve) direction to a distance 7m. ② moves to left to a distance of 25m along (-ve) axis. ③ moves to right.



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3) The moment at which ($a=0$)

let $a=0 \Rightarrow 6t - 18 = 0 \Rightarrow t = 3$ (s)

$s|_{t=3} = -9\,m$, $v|_{t=3} = -12\,m/s$.

Note when $a=0$, It changes the type of velocity due to accelerating and decelrating.

when $v=0$, It changes the direction of motion.

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4) at ($T=3$) Find the displacement and the total covered distance.

→ Displacement: $S_3 - S_0 = -9 - 0 = -9\,m$

$t=0$, $t=3$ (From last to first position).

→ Total travelled distance: From $(0 \to 1) = 7m$

$t=0 \to t=3$ From $(1 \to 3) = 16m$

Total distance $= 7 + 16 = 23\,m$.

5) at $t = 6$ (5) $\Rightarrow$ $S_6 = -18$ m.

→ Displacement: (S final $-$ S initial)

$= S_6 - S_0 = -18 - 0 = -18$

→ Total distance: $t(0 \to 1) = 7$ m

$t(1 \to 5) = 32$ m

$t(5 \to 6) = 7$ m

Total distance $= 7 + 7 + 32 = 46$ m.

→ Total traveled distance

$$= \left| S_{v=0} - S_0 \right| + \left| S_{final} - S_{v=0} \right|$$

OR

$$= \left| S_{v=0} - S_0 \right| + \left| S_{v_2} - S_{v_1} \right| + \left| S_{final} - S_{v=0} \right|$$

Note: Newoten's law of acceleration used only in the case of constant acceleration.

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Section : 45

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